# A Practical Approach for the FIFO Stack-Up Problem

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**Abstract.** We consider the FIFO Stack-Up problem which arises in delivery industry, where bins have to be stacked-up from conveyor belts onto pallets. Given $k$ sequences $q_1, \ldots, q_k$ of labeled bins and a positive integer $p$. The goal is to stack-up the bins by iteratively removing the first bin of one of the $k$ sequences and put it onto a pallet located at one of $p$ stack-up places. Each of these pallets has to contain bins of only one label, bins of different labels have to be placed on different pallets. After all bins of one label have been removed from the given sequences, the corresponding stack-up place becomes available for a pallet of bins of another label. The FIFO Stack-Up problem is computational intractable [4]. In this paper we introduce a graph model for this problem, which allows us to show a breadth first search solution. Our experimental study of running times shows that our approach can be used to solve a lot of practical instances very efficiently.

**Keywords:** combinatorial optimization, breadth first search solution, experimental analysis

## 1 Introduction

We consider the combinatorial problem of stacking up bins from a set of conveyor belts onto pallets. A detailed description of the practical background of this work is given in [2, 7]. The bins that have to be stacked-up onto pallets reach the palletizer on a conveyor and enter a *cyclic storage conveyor*, see Fig. 1. From the storage conveyor the bins are pushed-out to *buffer conveyors*, where they are queued. The equal-sized bins are picked-up by stacker cranes from the end of a buffer conveyor and moved onto pallets, which are located at some *stack-up places*. Often there is one buffer conveyor for each stack-up place. Automatic guided vehicles (AGVs) take full pallets from stack-up places, put them onto trucks and bring new empty pallets to the stack-up places.

The cyclic storage conveyor enables a smooth stack-up process irrespective of the real speed the cranes and conveyors are moving. Such details are unnecessary to compute an order in which the bins can be palletized with respect to the given number of stack-up places. For the sake of simplicity, we disregard the cyclic storage conveyor, and for the sake of generality, we do not restrict the number of

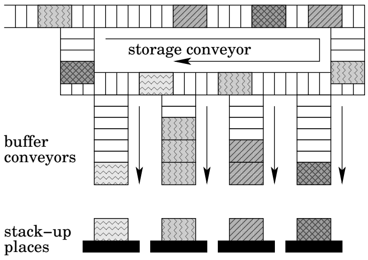


**Fig. 1.** A real stack-up system.

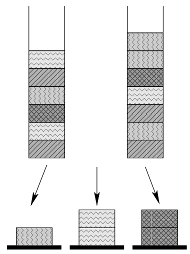

**Fig. 2.** A FIFO stack-up system.

stack-up places to the number of sequences. The number of sequences can also be larger than or less than the number of stack-up places. Fig. 2 shows a sketch of a simplified stack-up system with 2 buffer conveyors and 3 stack-up places.

From a theoretical point of view, we are given $k$ sequences $q_1, \ldots, q_k$ of bins and a positive integer $p$. Each bin is destined for exactly one pallet. The FIFO Stack-Up problem is to decide whether one can remove iteratively the bins of the $k$ sequences such that in each step only the first bin of one of the sequences will be removed and after each removal of a bin at most $p$ pallets are open. A pallet $t$ is called open, if at least one bin for pallet $t$ has already been removed from one of the given sequences, and if at least one bin for pallet $t$ is still contained in one of the remaining sequences. If a bin $b$ is removed from a sequence then all bins located behind $b$ are moved-up one position to the front.

Our model is the second attempt to capture important parameters necessary for an efficient and provable good algorithmic controlling of stack-up systems. The only theoretical model for stack-up systems known to us uses a random access storage instead of buffer queues. Many facts are known on the stack-up system model with random access storage, see [7–9], where complexity results as well as approximation and online algorithms are presented.

The FIFO Stack-Up problem is NP-complete even if the number of bins per pallet is bounded, but can be solved in polynomial time if the number $k$ of sequences or the number $p$ of stack-up places is fixed [4]. Dynamic programming solution and parameterized algorithms for the FIFO Stack-Up problem are shown in [5] and [6]. In this paper we introduce a graph model for this problem, the so called decision graph, which allows us to give a breadth first search solution of running time $\mathcal{O}(n^2 \cdot (m+2)^k)$, where $m$ represents the number of pallets and $n$ denotes the total number of bins in all sequences. Since for practical instance sizes this running time is too huge for computations, we used cutting technique on the decision graph by restricting to configurations such that the number of open pallets is at most some upper bound, which is increased by 5 until a solution is found. Our experimental study of running times shows that our approach can be used to solve a lot of practical instances on several thousand bins very efficiently.

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