Storage Controlled Pile-Up Systems, Theoretical Foundations

J. Rethmann* E. Wanke†

Abstract

This paper presents the theoretical foundations for controlling pile-up systems. A pile-up system consists of one or more stacker cranes picking up bins from a conveyor and placing them onto pallets with respect to costumer orders. The bins usually arrive at a conveyor from an orderpicking system.

We give a mathematical definition of the pile-up problem, define a data structure for control algorithms, introduce polynomial time algorithms for deciding whether a system can be blocked by making bad decisions, and show that the pile-up problem is in general NP-complete. For pile-up systems with a restricted storage capacity or with a fixed number of pile-up places the pile-up problem is proved to be solvable very efficiently.

Keywords: Computational analysis, complexity, discrete algorithms

*University of Düsseldorf, Department of Computer Science, D-40225 Düsseldorf, Germany, e-mail: rethmann@cs.uni-duesseldorf.de

†University of Düsseldorf, Department of Computer Science, D-40225 Düsseldorf, Germany, e-mail: wanke@cs.uni-duesseldorf.de

1 Introduction

A pile-up system usually consists of one or more stacker cranes which pick up bins from a conveyor and place them onto pallets. Vehicles take full pallets from pile-up places to trucks and bring new empty pallets to pile-up places. The conveyor at which the bins arrive is in general the back-end of an orderpicking system. To understand the problems involved when controlling pile-up systems we first sketch the operation mode of an orderpicking system. Most of our notations are adopted from [dK94].

A customer order consists of a list of articles. The ordered articles have to be packed for delivery into bins of equal size. An order is divided into several order lists, one for each bin. The order list contains the articles and their quantities to be picked from shelf racks into the bin. Empty bins are placed at the beginning of a conveyor. A barcode containing the order list number is attached to each empty bin. The barcode is used to recognize automatically a bin at control positions. A common size of the bins is 60 cm $\times$ 40 cm $\times$ 35 cm (l $\times$ w $\times$ h).

The conveyor transports the bins to the so-called picking stations. If some articles have to be picked at a picking station then the transportation system automatically pushes out the bin to a secondary conveyor. On the secondary conveyor the bin can be stopped without blocking the flow of bins on the main conveyor. The picker gets information about the articles to pick by a visual display unit. He picks the indicated articles from the shelves and put them into the bin. Having finished the picking, the bin is pushed back onto the main conveyor. The main conveyor transports the bin to the next picking station, etc.

The orderpicking systems we consider consist of a roller conveyor that cyclically connects all picking stations. A common conveyor speed is about 0.5 m/s. Figure 1.1 sketches a top view of such an orderpicking system. The picking stations and shelf racks are located in the so-called commission area. Each secondary conveyor has only a small storage capacity for approximately three or four bins at each picking station. If a bin can not be pushed out because the secondary conveyor is completely occupied by bins, then the bin is transported to the next picking station or is going on moving on the main conveyor until it reaches the same station again.

Such orderpicking systems are highly flexible in storage capacity as well as in picking capacity. The shelves can be divided into more slots to store more different products. More orders can be handled by simply bringing in more pickers. The capacity can be reduced by reducing the number of pickers and letting each picker handle several picking stations. Such orderpicking systems become more and more common by supply industry and warehouses in Germany and the Netherlands.

In [dK94], de Koster models acyclic orderpicking systems as a network of connected conveyor pieces and picking stations. The aim of the modeling is, for a particular design, to provide fast information on throughput times of bins, picker

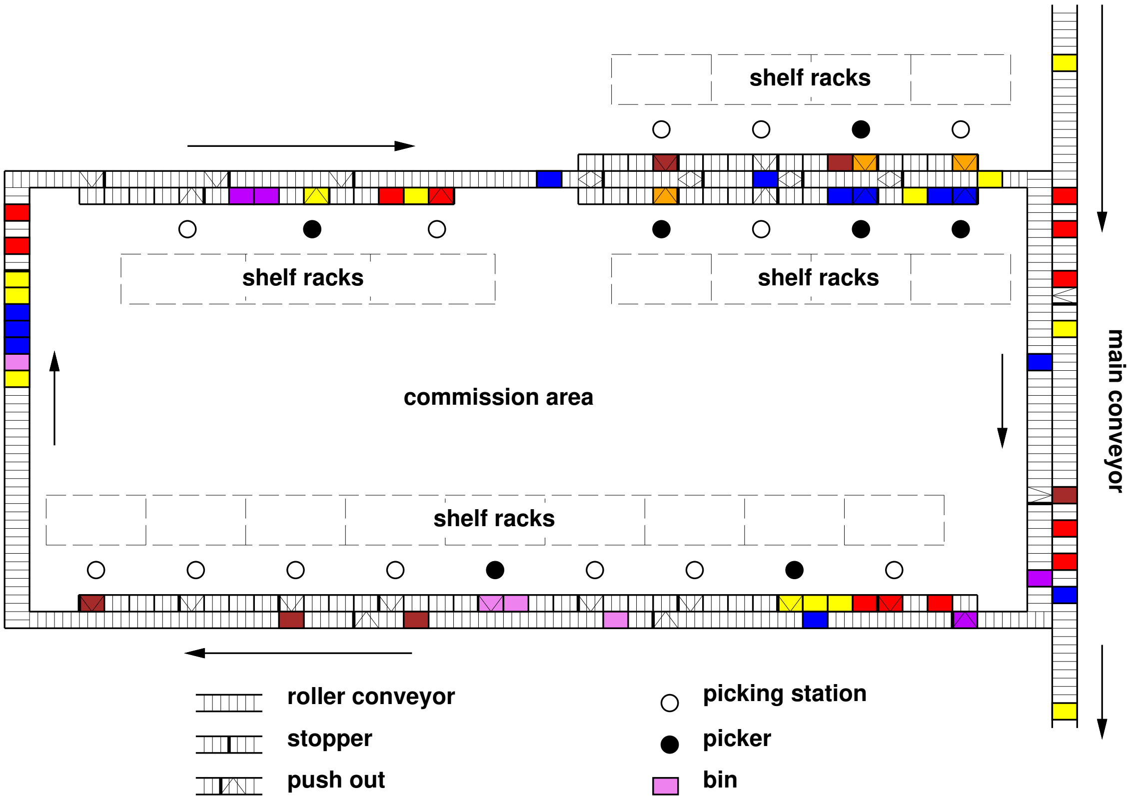

Figure 1.1: An orderpicking system.

utilization, and the average number of bins in the system. The analysis of the model is based on Jackson network analysis; see, for example, [GPK87]. The queues of the network can also be analyzed as stand-alone M/M/c queues[1] by standard methods; see, for example, [Kle75].

If not all picking stations are occupied by pickers then some bins possibly have to wait at the secondary conveyor of a picking station until some picker comes to the idle station and picks the articles. If the secondary conveyor is full the bins have to cycle at the main conveyor. Such waiting situations influence the overall throughput of the system and usually rearrange the bins. This rearrangement could cause some trouble when bins have to be placed for delivery onto pallets. All bins belonging to one costumer order have to be placed onto the same pallet if possible. If the number of bins of an order exceeds the capacity of a pallet then the order is divided into suborders of which each fit onto a pallet. A common

[1] Queueing systems can be charaterized using a standard shorthand form, $A/S/m$, known as Kendall's notation, where A specifies the interarrival-time distribution, S the service-time distribution and m the number of parallel (identical) machines. The standard symbol M for the interarrival-time and service-time describes the exponential (Markovian) distribution. A commonly employed scheduling discipline is FCFS (first-come-first-served), where jobs are served in the order of their arrival.

maximal capacity of a pallet is about 32 bins, 4 bins side by side and 8 bins one upon the other. Additionally, each pallet has to contain a minimal number of bins. A common minimal capacity of a pallet is about 28 bins. If orders or suborders contain less than the minimal number of bins then they will be combined and piled up on a so-called *mixed pallet*. Such a combination is only allowed if the involved orders are destined for the same geographical region.

The pile-up system is located at the end of an orderpicking system, where the bins have to be piled up onto pallets. Figure 1.2 sketches the top view of a pile-up system. The bins arrive the pile-up system on the main conveyor of the orderpicking system. At the end of the main conveyor the bins enter a cyclic storage conveyor. At the storage conveyor bins are pushed out to buffer conveyors, where they are queued. Each buffer conveyor B_i is associated with a primary pile-up place P_i and a secondary pile-up place Q_i. The bins in buffer conveyor B_i are destined for the pallets piled up on place P_i or Q_i. A stacker crane picks the stopped bin from the end of buffer conveyor B_i and place it onto the pallet located at primary place P_i. If a pallet is completely piled up, a vehicle brings it to a truck and moves a new empty pallet to the places.

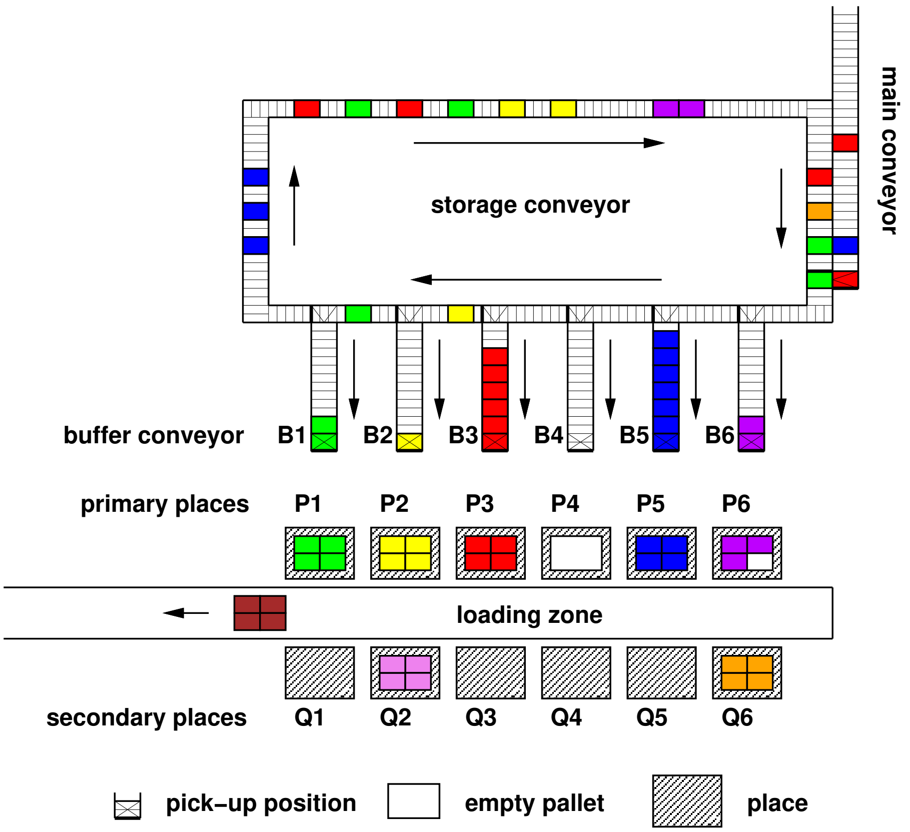

Figure 1.2: A pile-up system.

Unfortunately, the bins arrive the pile-up system not in a succession such that they can be placed one after the other onto pallets. If more places are needed then an incomplete pallet can temporarily be moved from a primary place P_i to a free secondary place Q_i. However, if all primary places and all secondary places are occupied by incomplete pallets and the storage conveyor is completely filled with bins not destined for the pallets on the places, then the pile-up system is blocked. To unblock the system, bins of the storage conveyor have to be temporarily put on the floor or bins from the main conveyor have to be manually picked and brought to the pallets. Due to the additional extra costs, a blocking situation is a nightmare for each distribution company.

Controlling a pile-up system means to make the right decision whenever a new pallet is piled up such that no blocking situation will arise. In this paper, we give the theoretical foundations for controlling such pile-up systems. The paper is organized as follows: In section 2, we introduce the main notations and give a formal definition of the pile-up problem. We are basically interested in how to control a pile-up system such that no blocking situation arise. The time needed to execute the suggested actions is more or less unimportant. Thus, in the formal definition, the possibility to use secondary places can be ignored. Secondary places can be handled like primary places, because the pallets on the places can be exchanged by the vehicle in the loading zone. Using buffer conveyors is also not essentially for the theoretical model, because bins can only be picked up from the end of the buffer conveyor and the bins in the buffer conveyor can not be rearranged. Thus stacker cranes could also pick up bins directly from the conveyor if this would technically be possible. In section 3, we define the decision graph and show that each algorithm for controlling a pile-up system has to solve an NP-complete problem. We also show that the question whether a system can be blocked by making bad decisions can be answered in polynomial time. In section 4, we consider restricted pile-up systems in which the capacity of the storage conveyor is restricted or the number of pile-up places is fixed. We show that even parallel algorithms exist for controlling restricted pile-up systems. In section 5, we discuss the possibility to form mixed pallets. Conclusions and discussions are given in section 6.

References

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