Moving bins from conveyor belts onto pallets using FIFO queues

Frank Gurski, Jochen Rethmann, and Egon Wanke

Abstract We study the combinatorial FIFO stack-up problem. In delivery industry, bins have to be stacked-up from conveyor belts onto pallets. Given k sequences $q_1, \ldots, q_k$ of labeled bins and a positive integer p, the goal is to stack-up the bins by iteratively removing the first bin of one of the k sequences and put it onto a pallet located at one of p stack-up places. Each of these pallets has to contain bins of only one label, bins of different labels have to be placed on different pallets. After all bins of one label have been removed from the given sequences, the corresponding place becomes available for a pallet of bins of another label. The FIFO stack-up problem is NP-complete in general. In this paper we show that the problem can be solved in polynomial time, if the number k of given sequences is fixed.

1 Introduction

We consider the combinatorial problem of stacking up bins from a set of conveyor belts onto pallets. This problem originally appears in *stack-up systems* that play an important role in delivery industry and warehouses. A detailed description of the practical background of this work is given in [2].

The bins that have to be stacked up onto pallets arrive at the stack-up system on the main conveyor of an order-picking system. At the end of the main conveyor they enter a *cyclic storage conveyor*, see Fig. 1. From the storage conveyor the bins are pushed out to *buffer conveyors*, where they are queued. The bins are picked-up by stacker cranes from the end of a buffer conveyor and moved onto pallets, which

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are located at some *stack-up places*. There is one buffer conveyor for each stack-up place. Automatic driven vehicles take full pallets from stack-up places, put them onto trucks and bring new empty pallets to the stack-up places.

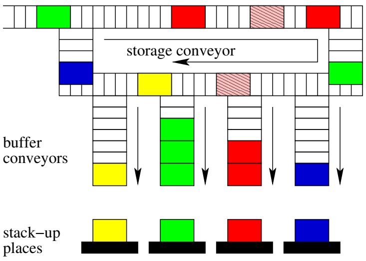

Fig. 1 A real stack-up system.

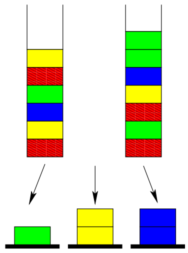

Fig. 2 A FIFO stack-up system.

In real life, the cyclic storage conveyor is necessary to enable a smooth stack-up process irrespective of the real speed the cranes and conveyors are moving. Such details are unnecessary to compute an order in which the bins can be palletized. So, in our model we neglect the cyclic storage conveyor, and the number of stack-up places is not correlated to the number of sequences. Logistic experiences over 30 years lead to such high flexible conveyor systems in delivery industry. So, we do not intend to modify the architecture of existing systems, but try to develop efficient algorithms to control them. Fig. 2 shows a sketch of a simplified stack-up system with 2 buffer conveyors and 3 stack-up places.

The FIFO stack-up problem has important practical applications. Many facts are known on stack-up systems that use a random access storage instead of buffer queues. The stack-up problem with random access storage is NP-complete, but can be solved efficiently if the storage capacity s or the number of stack-up places p is fixed. It remains NP-complete as shown in [5], even if the sequence contains at most 9 bins per pallet. A polynomial time *off-line* approximation algorithm is introduced in [5] that yields a processing that is optimal up to a factor bounded by $\log(p)$. In [6] the performances of some simple *on-line* stack-up algorithms are compared with optimal off-line solutions by a competitive analysis [1, 4].

The FIFO stack-up problem is NP-complete even if the number of bins per pallet is bounded [3]. In this paper we show by dynamic programming that the FIFO stack-up problem can be solved in polynomial time for a fixed number k of sequences.

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