Capital Budgeting Problems: A parameterized point of view

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Abstract A fundamental financial problem is budgeting. A firm is given a set of financial instruments $X = \{x_1, \ldots, x_n\}$ over a number of time periods T. Every instrument x_i has a return of r_i and for time period $t = 1, \ldots, T$ a price of $p_{t,i}$. Further for every time period t there is budget b_t. The task is to choose a portfolio X' from X such that for every time period $t = 1, \ldots, T$ the prices of the portfolio do not exceed the budget b_t and the return of the portfolio is maximized. We study the fixed-parameter tractability of the problem. For a lot of small parameter values we obtain efficient solutions for the capital budgeting problem. We also consider the connection to pseudo-polynomial algorithms.

1 Introduction

Capital budgeting can be regarded as a tool for maximizing a companys profit since most companies are able to manage only a limited number of projects at the same time. Capital budgeting problems have been introduced in [13] and [14]. See [16] for a survey on capital budgeting problems.

From a computational point of view the MAX MULTI-PERIOD CAPITAL BUDGETING is intractable. Since the problem is defined on inputs of various informations, in this paper we consider the fixed-parameter tractability for several parameterized versions of the problem. The idea behind fixed-parameter tractability is to split the complexity into two parts - one part that depends purely on the size of the input, and one part that depends on some *parameter* of the problem that tends to

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be small in practice. We also address the connection between these problems and pseudo-polynomial algorithms.

In this paper we use standard definitions for parameterized algorithms and pseudo-polynomial algorithms from the textbooks [1, 7, 8].

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