Algorithms for Controlling Palletizers

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Abstract Palletizers are widely used in delivery industry. We consider a large palletizer where each stacker crane grabs a bin from one of k conveyors and position it onto a pallet located at one of p stack-up places. All bins have the same size. Each pallet is destined for one customer. A completely stacked pallet will be removed automatically and a new empty pallet is placed at the palletizer. The FIFO STACK-UP problem is to decide whether the bins can be palletized by using at most p stack-up places. We introduce a digraph and a linear programming model for the problem. Based on these characterizations we give xp-algorithms and fpt-algorithms for various parameters, and approximation results for the problem.

1 Introduction

We consider the combinatorial problem of stacking up bins from a set of conveyor belts onto pallets. A detailed description of the practical background of this work is given in [2, 10]. The bins that have to be stacked up onto pallets reach the palletizer on a conveyor and enter a *cyclic storage conveyor*, see Fig. 1. From the storage conveyor the bins are pushed out to *buffer conveyors*, where they are queued. The equal-sized bins are picked-up by stacker cranes from the end of a buffer conveyor and moved onto pallets, which are located at some *stack-up places*. There is one buffer conveyor for each stack-up place. Automated guided vehicles (AGVs) take full pallets from stack-up places, put them onto trucks and bring new empty pallets to the stack-up places.

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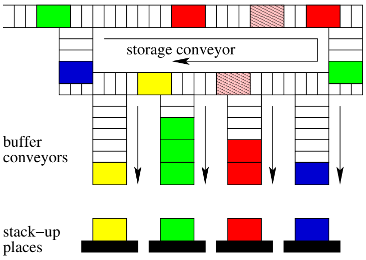

Fig. 1 A real stack-up system.

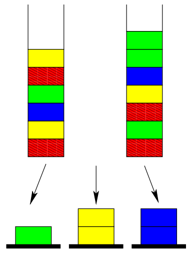

Fig. 2 A FIFO stack-up system.

The cyclic storage conveyor enables a smooth stack-up process irrespective of the real speed the cranes and conveyors are moving. Such details are unnecessary to compute an order in which the bins can be palletized with respect to the given number of stack-up places. For the sake of simplicity, we disregard the cyclic storage conveyor, and for the sake of generality, we do not restrict the number of stack-up places to the number of sequences. The number of sequences can also be larger than or less than the number of stack-up places. Fig. 2 shows a sketch of a simplified stack-up system with 2 buffer conveyors and 3 stack-up places.

From a theoretical point of view, we are given k sequences $q_1,\ldots,q_k$ of bins and a positive integer p. Each bin is destined for exactly one pallet. The FIFO STACK-UP problem is to decide whether one can remove iteratively the bins of the k sequences such that in each step only the first bin of one of the sequences will be removed and after each step at most p pallets are open. A pallet t is called open, if at least one bin for pallet t has already been removed from one of the given sequences, and if at least one bin for pallet t is still contained in one of the remaining sequences. If a bin b is removed from a sequence then all bins located behind b are moved-up one position to the front.

Our model is the second attempt to capture important parameters necessary for an efficient and provable good algorithmic controlling of stack-up systems. The only theoretical model for stack-up systems known to us uses a random access storage instead of buffer queues. We think that buffer queues model the real stack-up system more realistic than a random access storage. The FIFO STACK-UP problem seems to be not investigated by other authors up to know. However, many facts are known on the stack-up system model with random access storage, see [10, 11, 12].

The FIFO STACK-UP problem is NP-complete even if the number of bins per pallet is bounded [4]. In this paper we give algorithms for the FIFO STACK-UP problem based on a digraph model and a linear programming model.

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